Numerical Studies of Penetrative convection in Self-Magnetized Plasmas

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Introduction

In the numerical modeling of Core Collapse supernovae, various explosion mechanisms have been proposed to explain the morphology of system. In the initial stages of the supernova, the core of the progenitor begins to collapse until the matter within it is sufficiently dense to enter a degenerate state and becomes difficult to compress. This causes the collapsing core to bounce back, sending a shock wave propagating through the star. This initial bounce is too weak to drive the explosion however, leading to a variety of theories regarding how the shock is sufficiently strengthened to achieve the explosion, with the two most widely accepted being neutrino-driven convection and the standing accretion shock instability (SASI). In neutrino heating, the shock is powered by neutrinos emitted from the degenerate core and absorbed by material behind the shock . In this scenario, large scale fluid motion of material in the gain region, such as convection, is of particular interest, as it has critical implications on the dynamics of the explosion. In this work, we intend characterize the convection that arises in such scenarios, using the classic penetrative convection setup proposed by Hurlburt et al. (1986)

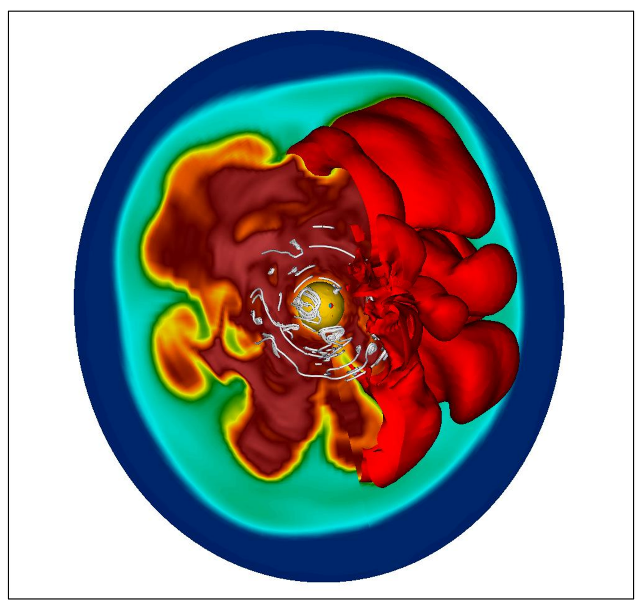

Figure 1: Numerical Simulation of a core collapse supernova and forming proto-neutron star. The explosion is 1600 km across at this point.

Numerical Model

Our numerical model is built within the Proteus version of the Flash code, a multidimensional, multiphysics hydrodynamic code. As we study the large scale movement of material, the most important equations for the dynamics of the system are the Euler equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{u}) = 0 \qquad (1)$$

$$\frac{\partial (\rho \mathrm{u})}{\partial \mathrm{t}} + \nabla \cdot (u \otimes (\rho u)) + \nabla p = 0 \qquad (2)$$

$$\frac{\partial E}{\partial \mathrm{t}} + \nabla \cdot (\mathrm{u}(\mathrm{E}+\mathrm{p})) = 0 \qquad (3)$$

Penetrative Convection

Our numerical study draws heavily on the penetrative convection configuration proposed by Hurlburt et al. (1986). In such a setup, a convectively unstable central layer of material is sandwiched by two layers stable against convection. These stability criteria are determined by relationship between two values, the radiative gradient , β_i, of the layer *i* and the adiabatic gradient of the domain, β_a, given by

$$\beta_i = F_T / K_i \qquad (4)$$

$$\beta_a = g / C_p \qquad (5)$$

Penetrative Convection (cont)

Where F_T is the total flux, (constant throughout the domain in the absence of motion) and K_i is the thermal conductivity of layer *i*, g is the constant gravitational acceleration, and C_p is the specific heat at constant pressure.

In the two layers that are stable against convection, the radiative gradient is held to be less than the adiabatic gradient, while in the unstable layer, the reverse is true.

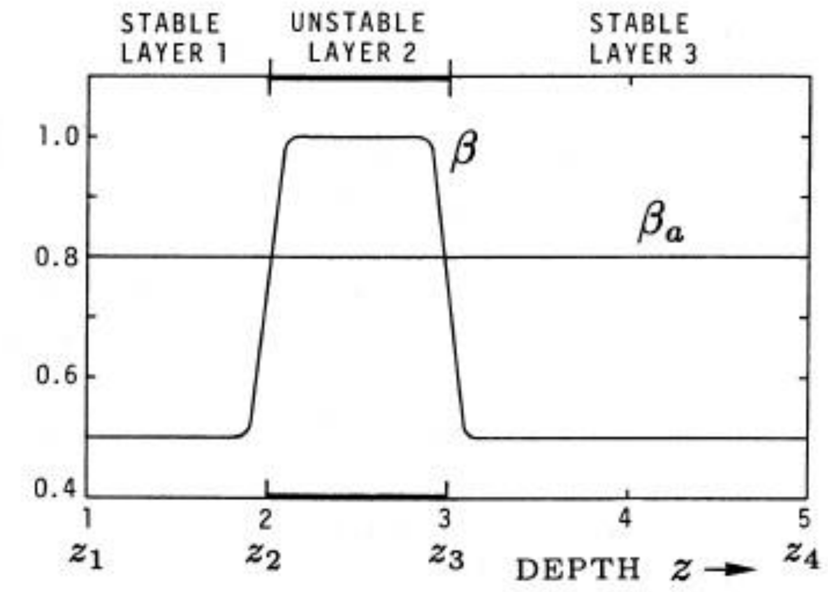

Figure 2: Profile of Radiative and Adiabatic gradients throughout the domain of the Hurlburt Penetrative Convection Problem. Note that our setup

The initial conditions for the Hurlburt setup follow a series of piecewise polytropes to describe the temperature, density, and pressure profiles throughout the domain. The polytropes for temperature (*T*), density (*ρ*), and pressure (*p*) are given by

$$T(z) = T_i + \frac{z - z_i}{K_i} \qquad (6)$$

$$\rho(z) = \rho_i \times \left(\frac{T(z)}{T_i}\right)^{m_i} \qquad (7)$$

$$p(z) = p_i \times \left(\frac{T(z)}{T_i}\right)^{m+1} \qquad (8)$$

Here *z* is the depth coordinate, m_i is the polytropic index, taken to be 3 in the stable layers and one in the central unstable layer. T_i , ρ_i, and p_i are constants set at the interfaces between layers in order to ensure the profiles of temperature, density, and pressure are continuous throughout the domain.

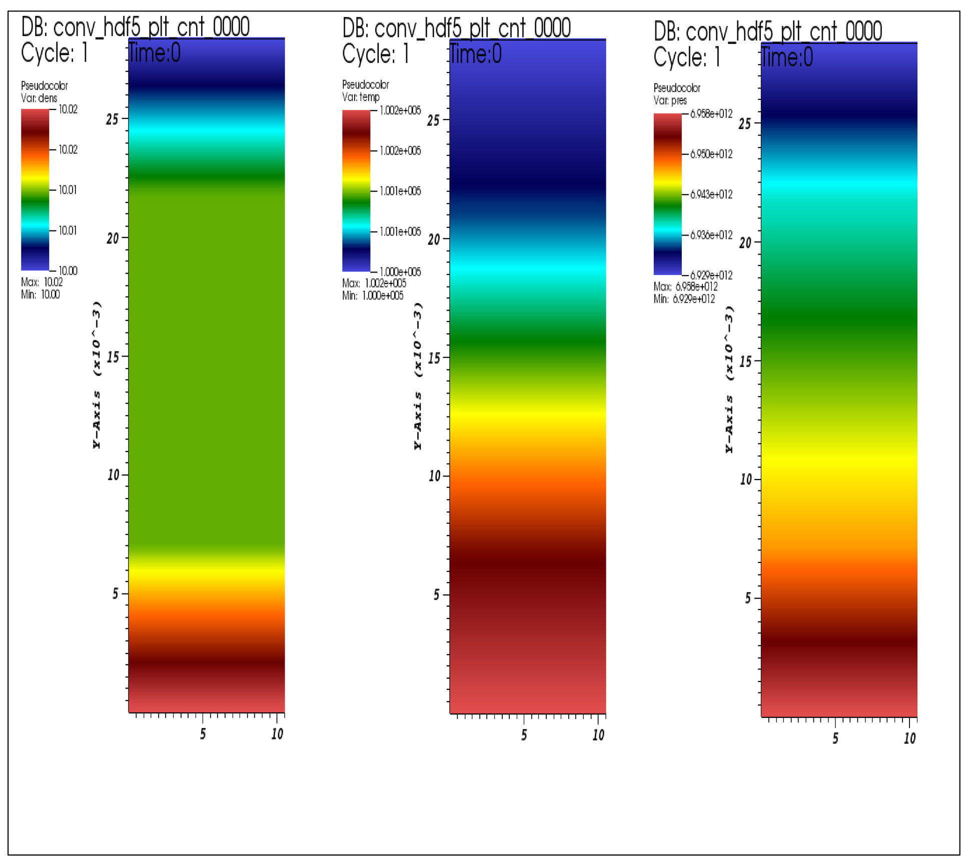

Figure 3: Initial distributions of density, temperature, and pressure in the Penetrative Convection setup.

Flux Integration

In order to characterize the behaviour of our penetrative convection study, several fluxes must be calculated at each time step; the convective, kinetic energy, radiative/conductive, and momentum fluxes, which are given below.

$$F_C = \int v\rho \cdot \left(\varepsilon + \frac{p}{\rho}\right)' d\Omega$$

$$F_K = \int v\rho \cdot \left(\frac{1}{2} v_i^{\,2}\right)' d\Omega$$

$$F_R = -\int K \partial t d\Omega$$

$$F_P = -\int v p' d\Omega$$

The above prime terms indicate the value is taken to be the perturbation from the mean of all the cell values within the integration; that is to say

$$A_i' = A_i - \sum_{i=1}^{n} \frac{A_i}{n}$$

Because the density, temperature and pressure are stratified vertically in our domain, we are interested in calculating these four fluxes in for each z value in our domain. Thus, for each row of cells on the z-axis of our discrete domain, we sweep across the x-coordinates and determine the value of the integrand for each cell, multiply that by the cross sectional area of the differential used, and divide that by the number of cells in each row.

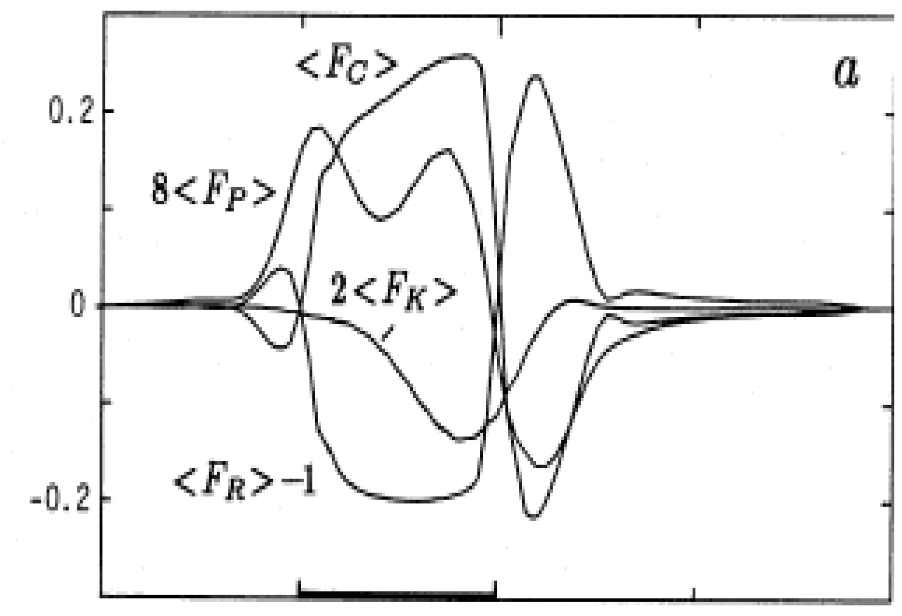

Figure 4: Time averaged flux values given oven depth from the Hurlburt problem. Note the penetration increased flux rates on both sides of the unstable layer interface.

Future Work

With the hydrodynamic and flux integral potions of the project complete, our focus will soon turn to the implementation of the Hurlburt setup within a magnetohydrodynamics setup, in order to allow us to characterize the generation and evolution of magnetic fields within the convectively unstable layer.

References

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